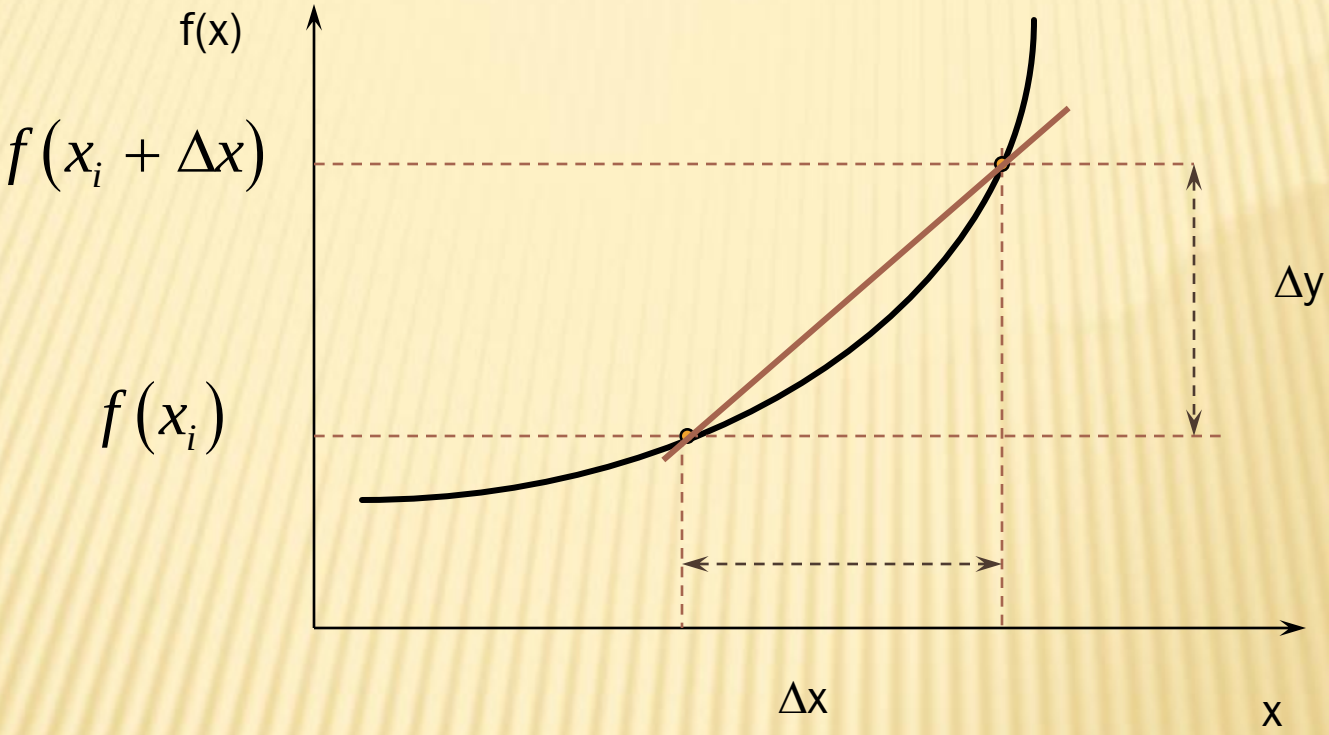
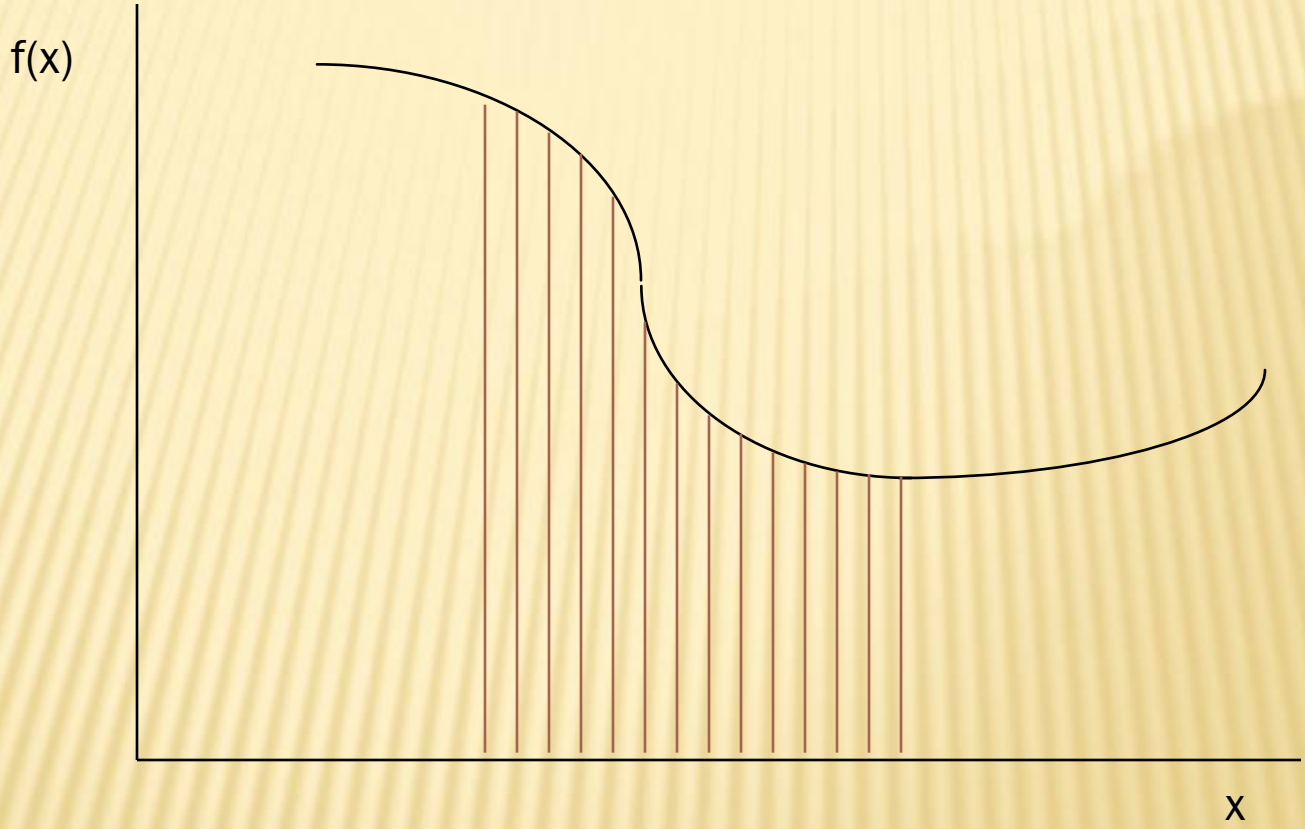
NUMERICAL INTEGRATION & DIFFERENTIATION



$$\frac{\Delta y}{\Delta x} = \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

INTEGRATION

- The inverse process of differentiation
- Dictionary definition of integrate - "*to bring together, as parts, into a whole; to unite; to indicate the total amount*"
- Mathematically, it is the total value or summation of $f(x)dx$ over a range of x. In fact the integration symbol is actually a stylized capital S intended to signify the connection between integration and summation.



$$I = \int_a^b f(x)dx$$

OVERVIEW

- Trapezoidal rule
- Simpson's Rules

NEWTON-COTES INTEGRATION

- Common numerical integration scheme
- Based on the strategy of replacing a complicated function or tabulated data with some approximating function that is easy to integrate

$$I = \int_a^b f(x)dx \cong \int_a^b f_n(x)dx$$

$$f_n(x) = a_0 + a_1 x + \ldots + a_n x^n$$

TRAPEZOIDAL RULE

- First of the Newton-Cotes closed integration formulas
- Corresponds to the case where the polynomial is a first order

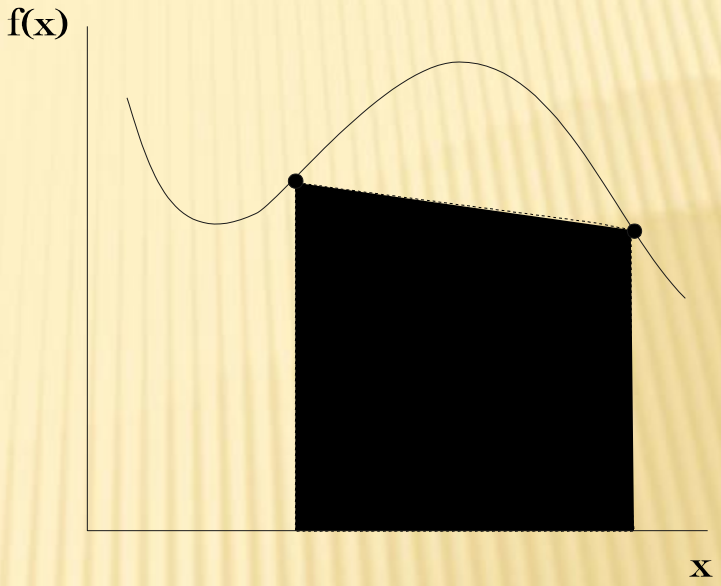
$$I = \int_a^b f(x)dx \cong \int_a^b f_1(x)dx$$

$$f_n(x) = a_0 + a_1 x$$

Trapezoidal Rule

$$I = \int_a^b f(x)dx \cong \int_a^b f_1(x)dx$$

$$f_n(x) = a_0 + a_1 x$$



A straight line can be represented as:

$$f_1(x) = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

Simpson's Rule:

$$\int_a^b f(x)\,dx = I$$

$$I \approx (b-a)/6 \{ f(a) + 4 f(a + b/2) + f(b) \}$$