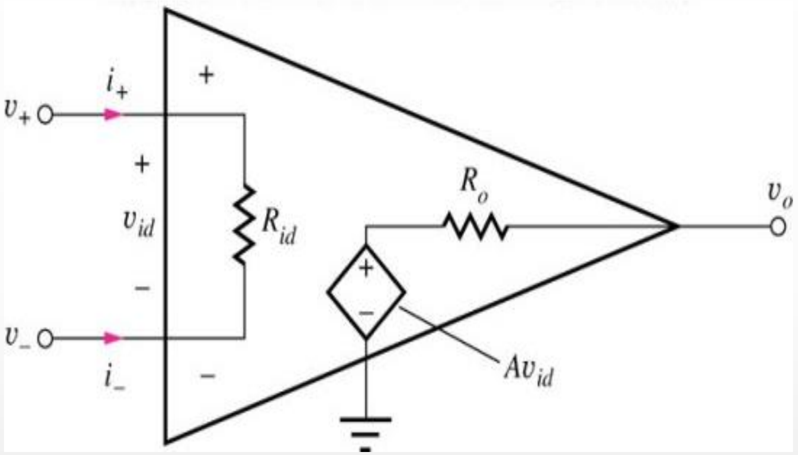
The Operational Amplifier

Why Study Op-Amps?

- Understand the "magic" of negative feedback and the characteristics of ideal op amps.
- Understand the conditions for non-ideal op amp behavior so they can be avoided in circuit design.
- Demonstrate circuit analysis techniques for ideal op amps.
- Characterize inverting, non-inverting, summing and instrumentation amplifiers, voltage follower and first order filters.
- Learn the factors involved in circuit design using op amps.
- Find the gain characteristics of cascaded amplifiers.
- Special Applications: The inverted ladder DAC and successive approximation ADC

Differential Amplifier Basics



Represented by:

A = open-circuit voltage gain

$v_{id} = (v^{+} - v^{-})$ = differential input signal voltage

R_{id} = amplifier input resistance

R_{o} = amplifier output resistance

The signal developed at the amplifier output is in phase with the voltage applied at the + input (non-inverting) terminal and 180° out of phase with that applied at the - input (inverting) terminal.

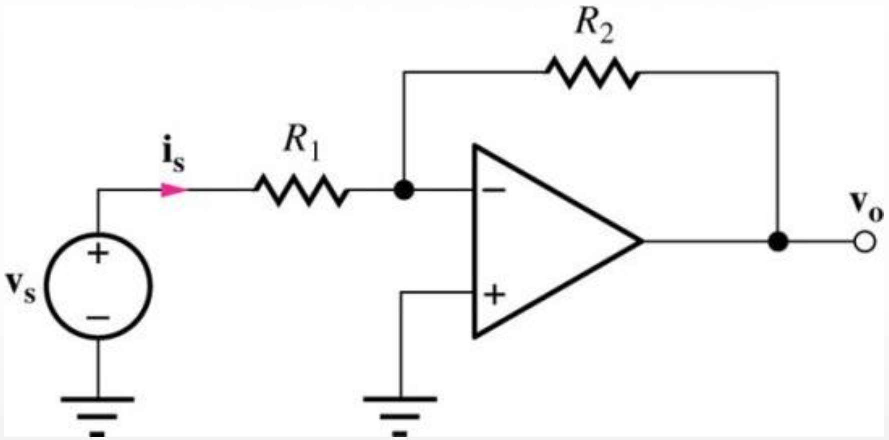
Ideal Operational Amplifier

- The "ideal" op amp is a special case of the ideal differential amplifier with infinite gain, infinite R_{id} and zero R_o .

$$v_{id} = \frac{v_o}{A} \quad \text{and} \quad \lim_{A\to\infty} v_{id} = 0$$

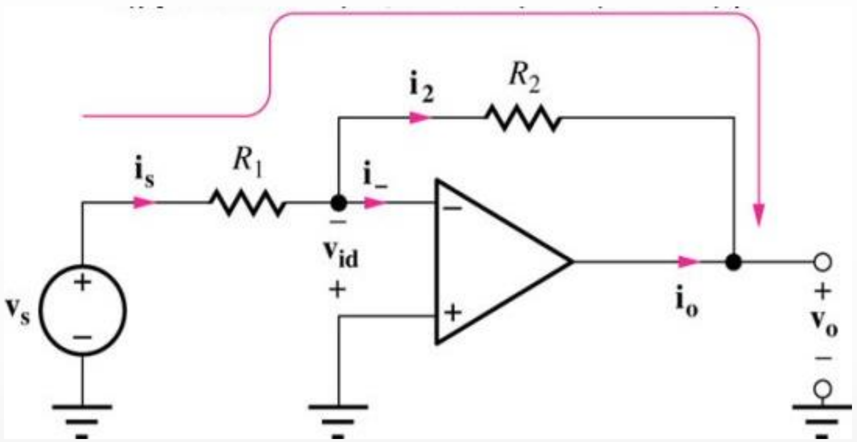
  - If A is infinite, v_{id} is zero for any finite output voltage.
  - Infinite input resistance R_{id} forces input currents i_+ and i_- to be zero.
- The ideal op amp operates with the following assumptions:
  - It has infinite common-mode rejection, power supply rejection, open-loop bandwidth, output voltage range, output current capability and slew rate
  - It also has zero output resistance, input-bias currents, input-offset current, and input-offset voltage.

The Inverting Amplifier: Configuration



- The positive input is grounded.
- A "feedback network" composed of resistors R_1 and R_2 is connected between the inverting input, signal source and amplifier output node, respectively.

Inverting Amplifier: Voltage Gain



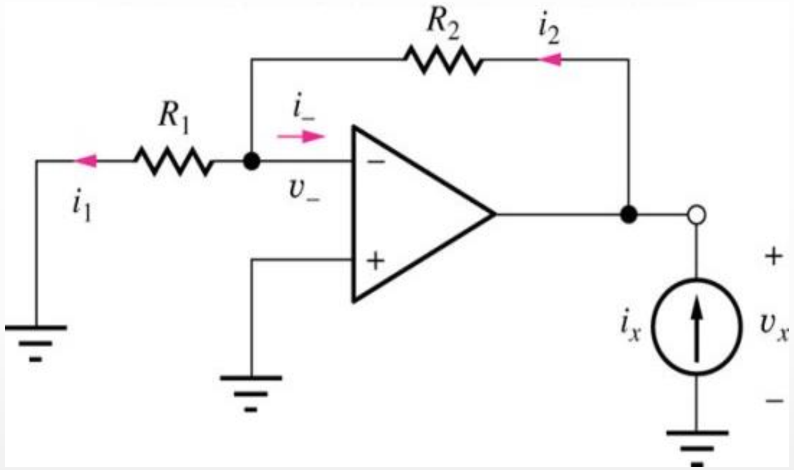
$$v_s - i_s R_1 - i_2 R_2 - v_o = 0$$

But $i_s = i_2$ and $v_- = 0$ (since $v_{id} = v_+ - v_- = 0$)

$$\therefore i_s = \frac{v_s}{R_1} \quad \text{and} \quad A_v = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

- The negative voltage gain implies that there is a 180° phase shift between both dc and sinusoidal input and output signals.
- The gain magnitude can be greater than 1 if $R_2 > R_1$
- The gain magnitude can be less than 1 if $R_1 > R_2$
- The inverting input of the op amp is at ground potential (although it is not connected directly to ground) and is said to be at **virtual ground.**

Inverting Amplifier: Input and Output Resistances



R_{out} is found by applying a test current (or voltage) source to the amplifier output and determining the voltage (or current) after turning off all independent sources. Hence, $v_s = 0$

$$v_x = i_2 R_2 + i_1 R_1$$

But $i_1 = i_2$

$$\therefore v_x = i_1 (R_2 + R_1)$$

$$R_{in} = \frac{v_s}{i_s} = R_1 \quad \text{since } v\_ = 0$$

Since $v_- = 0$, $i_1 = 0$. Therefore $v_x = 0$ irrespective of the value of i_x.

$$\therefore R_{out} = 0$$

The Inverting Amplifier: Example

- **Problem:** Design an inverting amplifier
- **Given Data:** A_v= 20 dB, R_{in} = 20kW,
- **Assumptions:** Ideal op amp
- **Analysis:** Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1.

$$A_v(dB) = 20\log_{10}(|A_v|), \quad \therefore |A_v| = 10^{40\text{dB}/20\text{dB}} = 100$$

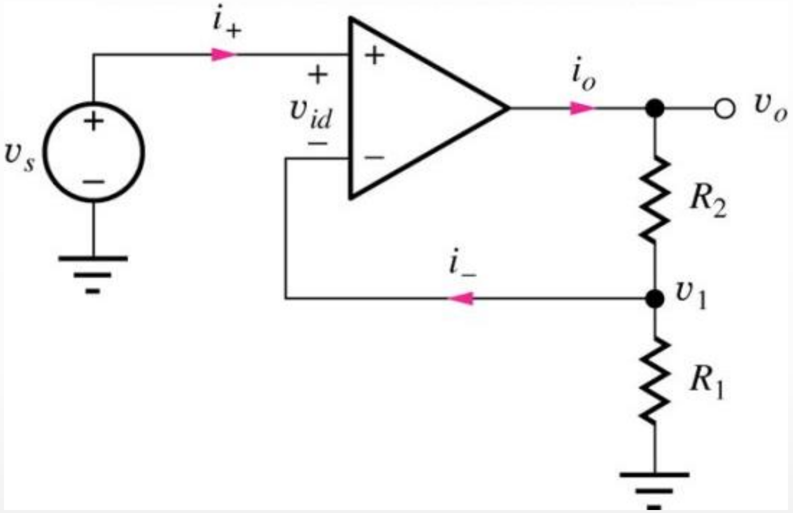
and A_v = -100

A minus sign is added since the amplifier is inverting.

$$R_1 = R_{in} = 20\text{k}\Omega$$

$$A_v = -\frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 2M\Omega$$

The Non-inverting Amplifier: Configuration



- The input signal is applied to the non-inverting input terminal.
- A portion of the output signal is fed back to the negative input terminal.
- Analysis is done by relating the voltage at v_1 to input voltage v_s and output voltage v_o.

Non-inverting Amplifier: Voltage Gain, Input Resistance and Output Resistance

Since $i_-=0$ $\quad v_1 = v_o \frac{R_1}{R_1+R_2}$ and $\quad v_s - v_{id} = v_1$

But $v_{id}=0$ $\quad \therefore v_s = v_1$

$$v_o = v_s \frac{R_1+R_2}{R_1}$$

$$\therefore A_v = \frac{v_o}{v_s} = \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$R_{in} = \frac{v_s}{i_+} = \infty$$

Since $i_+=0$

R_{out} is found by applying a test current source to the amplifier output after setting $v_s = 0$. It is identical to the output resistance of the inverting amplifier i.e. $R_{out} = 0$.

Non-inverting Amplifier: Example

- **Problem:** Determine the output voltage and current for the given non-inverting amplifier.
- **Given Data:** R_1= 3kW, R_2 = 43kW, v_s= +0.1 V
- **Assumptions:** Ideal op amp
- **Analysis:**

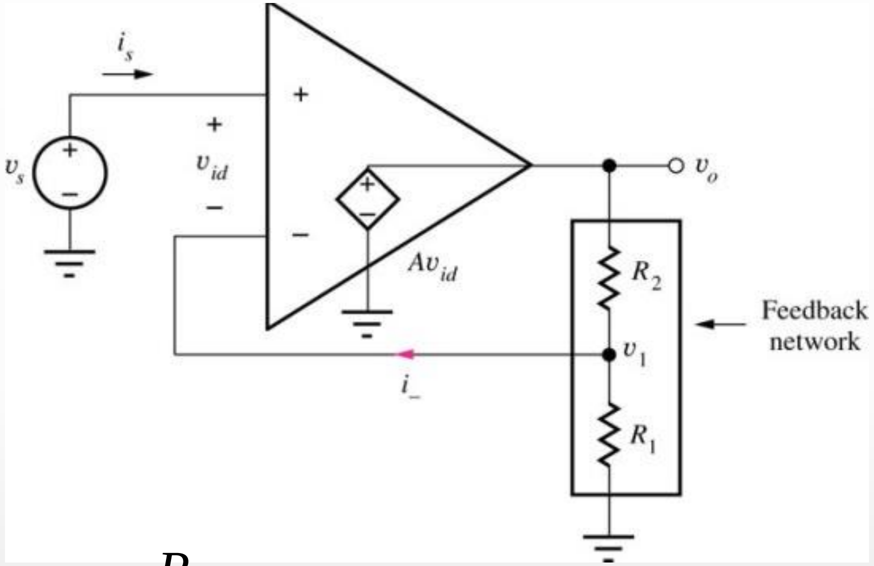
$$A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{43\mathrm{k}\Omega}{3\mathrm{k}\Omega} = 15.3$$

$$\mathrm{v_O} = A_v \mathrm{v_S} = (15.3)(0.1\mathrm{V}) = 1.53\mathrm{V}$$

Since $i_- = 0$,

$$\mathrm{i_O} = \frac{\mathrm{v_O}}{R_2 + R_1} = \frac{1.53\mathrm{V}}{43\mathrm{k}\Omega + 3\mathrm{k}\Omega} = 33.3\mu\mathrm{A}$$

Finite Open-loop Gain and Gain Error



$$v_1 = \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

$$\beta = \frac{R_1}{R_1 + R_2}$$ is called the **feedback factor.**

$$v_o = Av_{id} = A(v_s - v_1) = A(v_s - \beta v_o)$$

$$A_v = \frac{v_o}{v_s} = \frac{A}{1 + A\beta}$$

$A\beta$ is called loop gain.

For $A\beta >> 1$,

$$A_v \cong \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

This is the "ideal" voltage gain of the amplifier. If Ab is <u>not</u> >>1, there will be "Gain Error".

Gain Error

- Gain Error is given by

  GE = (ideal gain) - (actual gain)

  For the non-inverting amplifier,

$$GE = \frac{1}{\beta} - \frac{A}{1+A\beta} = \frac{1}{\beta(1+A\beta)}$$

- Gain error is also expressed as a fractional or percentage error.

$$FGE = \frac{\frac{1}{\beta} - \frac{A}{1+A\beta}}{\frac{1}{\beta}} = \frac{1}{1+A\beta} \cong \frac{1}{A\beta}$$

$$PGE \cong \frac{1}{A\beta} \times 100\%$$

Gain Error: Example

- **Problem:** Find ideal and actual gain and gain error in percent
- **Given data:** Closed-loop gain of 100,000, open-loop gain of 1,000,000.
- **Approach:** The amplifier is designed to give ideal gain and deviations from the ideal case have to be determined. Hence,

$$\beta = \frac{1}{10^5}$$

Note: R_1 and R_2 aren't designed to compensate for the finite open-loop gain of the amplifier.

- **Analysis:**

$$A_v = \frac{A}{1+A\beta} = \frac{10^6}{1+\frac{10^6}{10^5}} = 9.09x10^4$$

$$\text{PGE} = \frac{10^5 - 9.09x10^4}{10^5} \times 100\% = 9.09\%$$

Output Voltage and Current Limits

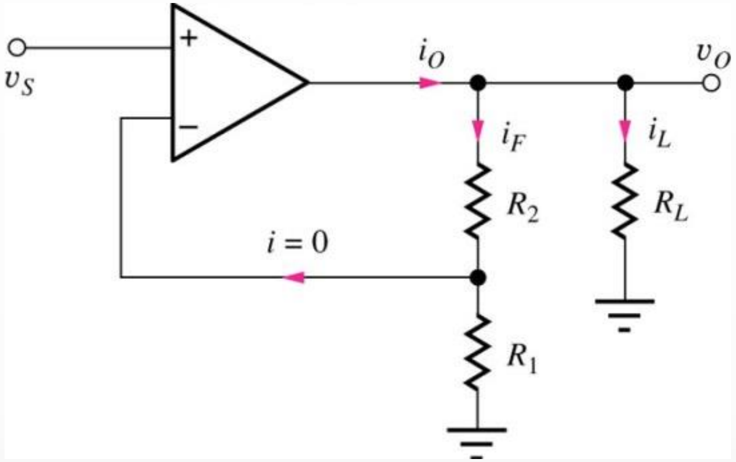
Practical op amps have limited output voltage and current ranges.

Voltage: Usually limited to a few volts less than power supply span.

Current: Limited by additional circuits (to limit power dissipation or protect against accidental short circuits).

The current limit is frequently specified in terms of the minimum load resistance that the amplifier can drive with a given output voltage swing. Eg:

$$|i_O| = \frac{5\text{V}}{500\Omega} = 10\text{mA}$$



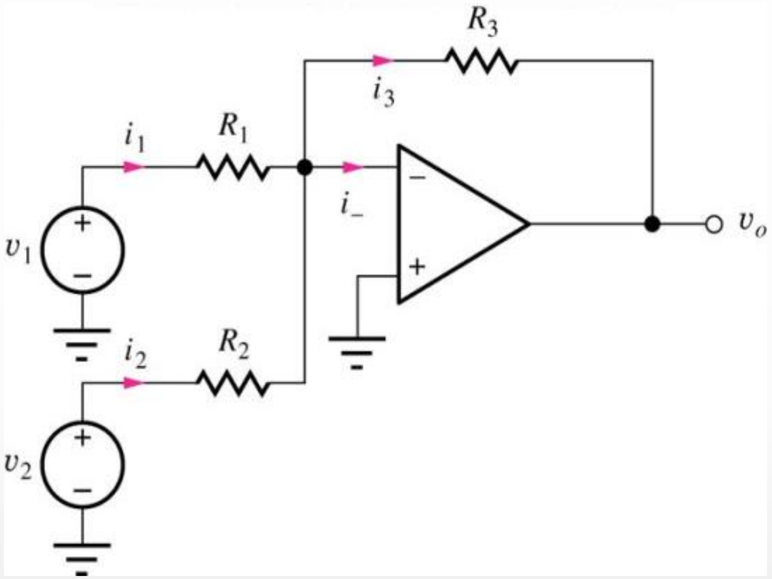
$$i_O = i_L + i_F = \frac{v_O}{R_L} + \frac{v_O}{R_2 + R_1} = \frac{v_O}{R_{EQ}}$$

$$R_{EQ} = R_L \| (R_1 + R_2)$$

For the inverting amplifier,

$$R_{EQ} = R_L \| R_2$$

The Summing Amplifier



Since the negative amplifier input is at virtual ground,

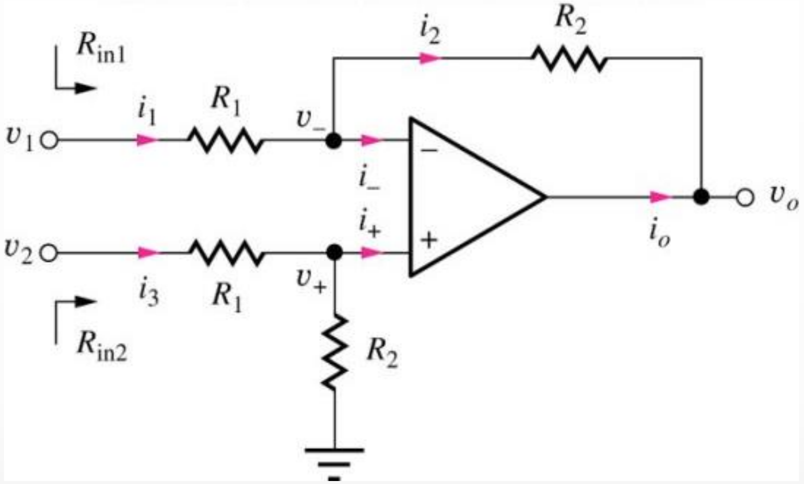
$$i_1 = \frac{v_1}{R_1} \quad i_2 = \frac{v_2}{R_2} \quad i_3 = -\frac{v_o}{R_3}$$

Since $i_- = 0$, $i_3 = i_1 + i_2$,

$$\therefore v_o = -\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_2$$

- Scale factors for the 2 inputs can be independently adjusted by the proper choice of R_2 and R_1.
- Any number of inputs can be connected to a summing junction through extra resistors.
- This circuit can be used as a simple digital-to-analog converter. This will be illustrated in more detail, later.

The Difference Amplifier



$$v_o = v_- - i_2 R_2 = v_- - i_1 R_2$$

$$= v_- - \frac{R_2}{R_1}(v_1 - v_-) = \left(\frac{R_1 + R_2}{R_1}\right) v_- - \frac{R_2}{R_1} v_1$$

Also, $$v_+ = \frac{R_2}{R_1 + R_2} v_2$$

Since $v_- = v_+$ $$v_o = -\frac{R_2}{R_1}(v_1 - v_2)$$

For $R_2 = R_1$ $$v_o = -(v_1 - v_2)$$

- This circuit is also called a differential amplifier, since it amplifies the difference between the input signals.
- R_{in2} is series combination of R_1 and R_2 because i_+ is zero.
- For v_2=0, R_{in1}= R_1, as the circuit reduces to an inverting amplifier.
- For general case, i_1 is a function of both v_1 and v_2.

Difference Amplifier: Example

- **Problem:** Determine v_o
- **Given Data:** R_1= 10kW, R_2 =100kW, v_1=5 V, v_2=3 V
- **Assumptions:** Ideal op amp. Hence, $v_- = v_+$ and $i_- = i_+ = 0$.
- **Analysis:** Using dc values,

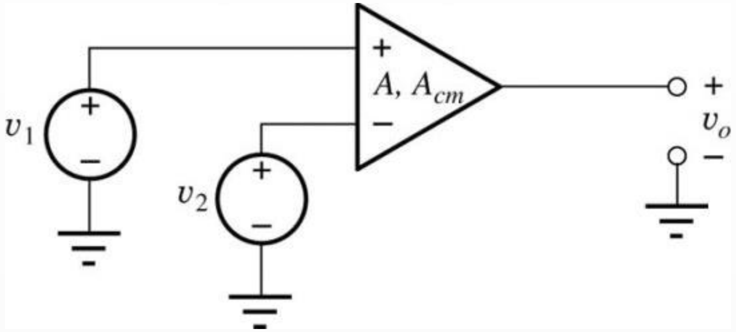
$$A_{dm} = -\frac{R_2}{R_1} = -\frac{100\text{k}\Omega}{10\text{k}\Omega} = -10$$

$$V_o = A_{dm}\left(V_1 - V_2\right) = -10(5-3)$$

$$V_o = -20.0 \text{ V}$$

Here A_{dm} is called the "differential mode voltage gain" of the difference amplifier.

The Common-Mode Rejection Ratio (CMRR)



A (or A_{dm}) = differential-mode gain

A_{cm} = common-mode gain

v_{id} = differential-mode input voltage

v_{ic} = common-mode input voltage

$$v_1 = v_{ic} + \frac{v_{id}}{2} \qquad v_2 = v_{ic} - \frac{v_{id}}{2}$$

A real amplifier responds to signal common to both inputs, called the common-mode input voltage (v_{ic}). In general,

$$v_o = A_{dm}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

$$v_o = A_{dm}(v_{id}) + A_{cm}(v_{ic})$$

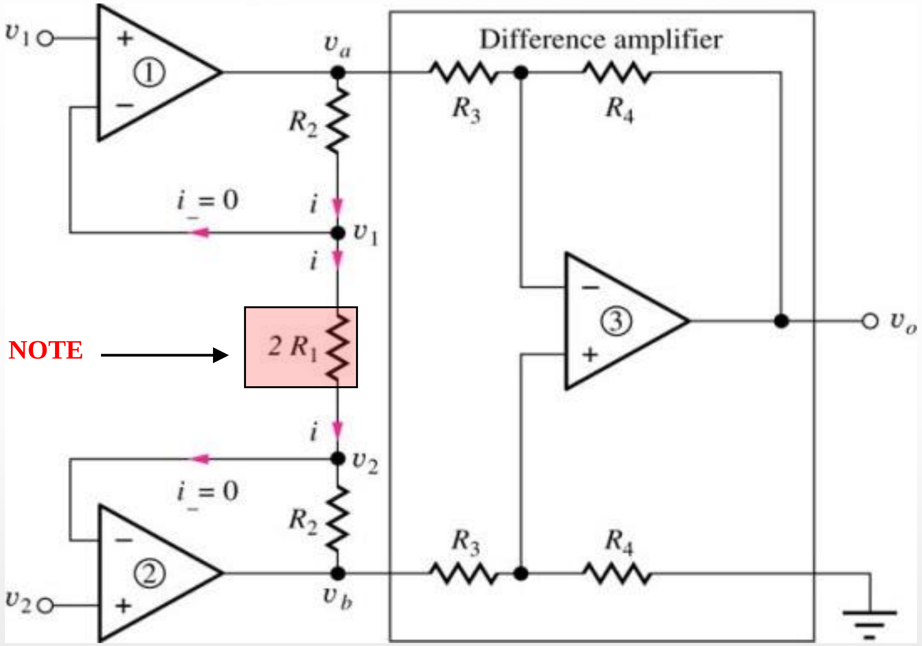
An ideal amplifier has $A_{cm} = 0$, but for a real amplifier,

$$v_o = A_{dm}\left(v_{id} + \frac{A_{cm} v_{ic}}{A_{dm}}\right) = A_{dm}\left(v_{id} + \frac{v_{ic}}{\text{CMRR}}\right)$$

$$\text{CMRR} = \left|\frac{A_{dm}}{A_{cm}}\right|$$

and $\text{CMRR(dB)} = 20\log_{10}(\text{CMRR})$

Instrumentation Amplifier



$$v_o = -\frac{R_4}{R_3}(v_a - v_b)$$

$$v_a - iR_2 - i(2R_1) - iR_2 = v_b$$

$$i = \frac{v_1 - v_2}{2R_1}$$

$$\therefore v_o = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)(v_1 - v_2)$$

Combines 2 non-inverting amplifiers with the difference amplifier to provide higher gain and higher input resistance.

Ideal input resistance is infinite because input current to both op amps is zero. The CMRR is determined only by Op Amp 3.

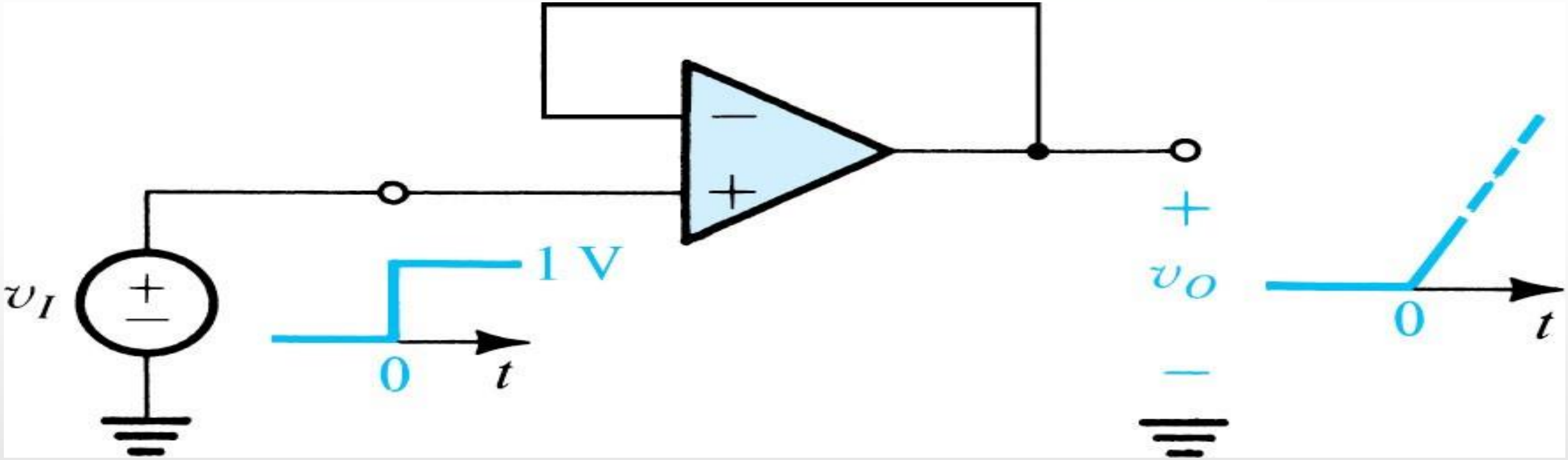
Instrumentation Amplifier: Example

- **Problem:** Determine V_o
- **Given Data:** $R_1 = 15$ kW, $R_2 = 150$ kW, $R_3 = 15$ kW, $R_4 = 30$ kW $V_1 = 2.5$ V, $V_2 = 2.25$ V
- **Assumptions:** Ideal op amp. Hence, $v_- = v_+$ and $i_- = i_+ = 0$.
- **Analysis:** Using dc values,

$$A_{dm} = -\frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) = -\frac{30\text{k}\Omega}{15\text{k}\Omega}\left(1+\frac{150\text{k}\Omega}{15\text{k}\Omega}\right) = -22$$

$$V_O = A_{dm}(V_1 - V_2) = -22(2.5-2.25) = -5.50\text{V}$$

The SlewRate



A unity-gain follower with a large step input. Since the output voltage cannot change immediately, a large differential voltage appears between the op-amp input terminals.