Discrete Mathematics
CC318

Propositional Logic

A *proposition* is a declarative statement that's either TRUE or FALSE (but not both).

What's a proposition?

Statements (Propositions)	Sentences (Not Propositions)
3 + 2 = 32	Bring me coffee!
CS173 is Bryan's favorite class.	CS173 is her favorite class.
Every cow has 4 legs.	3 + 2
There is another life in the universe.	Do you like Cake?

Propositional Logic - negation

Suppose *p* is a statement (proposition).

The *negation* of *p* is written $\neg p$ and has meaning:

"It is not the case that *p*."

- Ex. :

CC318 is NOT Ahmed's favorite class.

Truth table for negation:

p	$\neg p$
T	F
F	T

Notice that $\neg p$ is a Statement (proposition)!

Propositional Logic - conjunction

Conjunction corresponds to English "and."

$p \wedge q$ is true exactly when p and q are both true.

- Ex. Laila is curious AND clever.

Truth table for conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional Logic - disjunction

Disjunction corresponds to English "or."
$p \vee q$ is when p or q (or both) are true.

- Ex. Michael is brave OR nuts.

Truth table for disjunction:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Propositional Logic - implication

Implication: $p \to q$ corresponds to English "if p then q," or "p implies q."

- If it is raining then it is cloudy.
- If there are 200 people in the room, then I am a flying eagle.
- If p then 2+2=4.

Truth table for implication:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Propositional Logic - logical equivalence

How many different logical connectives could we define?

16

How many different logical connectives do we need?

??

To answer, we need the notion of "logical equivalence."

p is *logically equivalent* to *q* if their truth tables are the same. We write $p \equiv q$.

Propositional Logic - logical equivalence

Challenge: Try to find a proposition that is equivalent to $p \to q$, but that uses only the connectives $\neg$, $\wedge$, and $\vee$.

If you study hard, you pass the course!

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$q \vee \neg p$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Propositional Logic - proof of 1 famous ≡

I could say "prove a law of distributivity."

Distributivity: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
[illegible]	T	F	F	F	T	F	F
[illegible]	[illegible]	T	F	F	F	T	F
[illegible]	[illegible]	F	F	F	F	F	F

All truth assignments for *p*, *q*, and *r*.

Propositional Logic - special definitions

Contrapositives: $p \rightarrow q$ and $\neg q \rightarrow \neg p$

- Ex. "If it is noon, then I am hungry."

  "If I am not hungry, then it is not noon."

Converses: $p \rightarrow q$ and $q \rightarrow p$

- Ex. "If it is noon, then I am hungry."

  "If I am hungry, then it is n

Inverses: $p \rightarrow q$ and $\neg p \rightarrow \neg q$

- Ex. "If it is noon, then I am hung

  "If it is not noon, then I am not

One of these things is not like the others.

Hint: In one instance, the pair of propositions is equivalent.

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Propositional Logic - 2 more defn…

A *tautology* is a proposition that's always TRUE.

A *contradiction* is a proposition that's always FALSE.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Propositional Logic - say a bit…

This week we're using propositional logic as a foundation for formal proofs.

Propositional logic is also the key to writing good code…you can't do any kind of conditional (if) statement without understanding the condition you're testing.

All the logical connectives we've discussed are also found in hardware and are called "gates."

We'll talk about more applications next time.

Propositional Logic - for next time…

I will assume you know the definitions of the “famous” logical equivalences found on Rosen page 24. Bring a cheat sheet of them to class.